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The transit of Venus, 8 June 2004: a teachers' guide to finding the Earth–Sun distance

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Abstract

Transits of Venus have been observed since the 17th century and were soon recognized as a way of determining the distance from the Earth to the Sun. But just how can this be done? There are in fact four methods and this article examines them in turn, making clear how Venus will appear to move, what observations are required, the calculations that follow and the relative merits of each method.

Introduction

On 8 June 2004, Venus will pass across the face of, or transit, the Sun, something that has not happened since December 1882. This will be repeated eight years later on 6 June 2012. This article describes the transit and shows how observations can be used to find the distance of the Earth from the Sun, which forms the baseline from which ultimately the size and age of the universe are found.

The transit will be seen everywhere the Sun is above the horizon from 05:00 hrs to 11:30 UTC, which includes most of Africa and Europe, the Middle East and India. People in the Far East and Australia will see the start and miss the end, while those in the eastern USA will miss the start and see the end. The 2012 transit is not visible from Europe. It will start over the Pacific and end over the Far East. Figure 1 shows how different places on Earth see different views of the event, which forms the basic idea behind all methods for using the transit to find the scale of the solar system.

This article is particularly aimed at teachers who are curious to know what lies behind the brief statement (see e.g. the article by Arkan Simaan in this issue of *Physics Education*) that by studying the transit of Venus it is possible to make accurate measurements of the distance to the Sun.

The transit of 1639 was the first recorded observation of a transit of Venus (Stephenson 2004). It was predicted by Jeremiah Horrocks on the basis of earlier predictions by Johannes Kepler, and seen by Horrocks and Crabtree. In the early 18th century Halley proposed using transits of Venus as a method of finding the size of the solar system. As a result, many scientific expeditions were sent to observe the transits of 1761 and 1769 and again in 1874 and 1882. However, by the latter half of the 19th century more accurate methods were available, and today radar is used to measure the distance to Venus directly.

Since the early 16th century and the work of Copernicus, we have built an increasingly accurate model of our solar system. All observations of the planets and Sun made from Earth are either of angles or angular velocities. In order to fix the scale of the solar system it is necessary to calibrate an angle or angular velocity in terms of a physical distance or velocity, measured in km or km h^{-1}. During a transit of Venus this can be done using four methods. They are:

1. Observing the different apparent paths of Venus across the face of the Sun seen from

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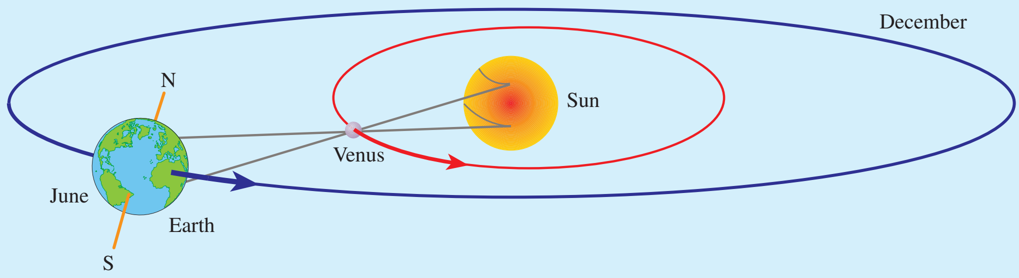

Figure 1. A view of Venus passing between the Earth and the Sun as it overtakes the Earth on the inside orbit. The different apparent paths of Venus across the Sun are shown for two sites at different latitudes, one in the North and one in the South. The effect is greatly exaggerated.

different widely spaced latitudes, as illustrated in figure 1.

2. Measuring the rate (angular velocity) at which Venus crosses the face of the Sun. The rate is the vector sum of the orbital angular velocity of Venus and the Earth and the angular velocity of the observing site due to the Earth's daily rotation.
3. Comparing the absolute times of Venus's contact with the Sun's limb, seen from different places on Earth (Delisle's method).
4. Comparing the total time for Venus to cross the Sun as seen from different latitudes, which depends on both the curvature of the Sun's limb and the distance moved by the observer due to the Earth's rotation (Halley's method).

Method 4 is the method proposed by Halley in his paper published in the *Proceedings of the Royal Society* in 1716. Its great advantage is that it requires only an accurate measurement of the time taken for the transit and not the absolute times of events. In other words, although the latitude and longitude must be known, they do not have to be known very accurately, as was the case in Halley's time. In 1724 Delisle visited Halley and suggested method 3.

The four methods are described in detail in sections 1 to 4 below and their main advantages and disadvantages are summarized in table 1. Methods 2, 3 and 4 all require the daily rotation of the Earth to be taken into account.

The summary at the end of this paper shows how all the data needed to find the scale of the solar system can be obtained. The data used here are taken from the *Explanatory Supplement to the Astronomical Almanac* (1992), while predictions for the 2004 transit are given in *The Astronomical Almanac 2004*.

Other methods for finding the distance

For the sake of completeness, two other methods for measuring the scale of the solar system, both relying on the speed of light, should be mentioned. In 1676 Ole Römer reported that as the separation of the Earth and Jupiter increases, as they both orbit the Sun, eclipses of the Jovian satellites occur progressively later. He correctly attributed this to the finite speed of light. This was not widely believed until 1729 when Bradley announced his discovery of aberration, which causes the displacement of a star's observed position because the Earth's orbital velocity is a measurable fraction of the speed of light. The total effect is at most 40 seconds of arc (hereafter denoted $40''$) but it proved for the first time that the Earth moves through space. Both methods provide a means of fixing the size of the solar system provided the velocity of light is known.

Frequency of transits

The plane of Venus's orbit is inclined at an angle of $3^\circ.394$ to the plane of the Earth's orbit, so although Venus overtakes the Earth on the inside orbit every 1.6 years as they both circle the Sun, a transit only occurs if Venus and the Earth are both within about $1^\circ.7$ of the point where their orbital planes intersect, placing the Earth, Venus and the Sun in a straight line. The range of $\pm 1^\circ.7$ arises because of the Sun's finite angular diameter. The intersection of the two planes containing the orbits

Table 1. The main advantages and disadvantages of the four methods of calibrating the scale of the solar system.

Method	Type of observations required	Observing sites required	Ease of observation	Ease of understanding	Accuracy
1	Position of Venus on Sun	2	Difficult	Easy	Low
2	Position of Venus on Sun and duration of transit	1	More difficult	Hardest	Lowest
3	Absolute times of start and end of transit	3	Easy	Harder	Highest
4	Duration of transit	3	Easiest	Harder	Highest

is referred to as the line of nodes. Around the orbit there are two nodes so transits can only happen in either early June or six months later in early December. Venus always crosses the Sun from East to West. In December at the ascending node, it heads slightly north and in June at the descending node, south. (This is very well explained in the Paper Plate demonstration featured in Frontline, page 226 in this issue.)

At present, pairs of transits eight years apart occur every 243 years at each of the nodes. The pairs of transits occurring at the December ascending node are offset in time from the pairs at the June descending node in such a way that it is the second transit of the pair that lies midway (121.5 years) between the first of successive pairs of June transits. This behaviour is summarized in figure 2.

The occurrence of pairs of transits, separated by eight years, arises from an unusual circumstance that will prevail for at least the next 1200 years. The first transit of the pair occurs when Venus and the Earth are about 1° beyond the node. This allows a second transit to occur eight years later, when Venus and the Earth are about the same distance before the same node. If transits were taking place exactly at the node there would be no second transit eight years later and transits would occur regularly at alternate nodes every 121.5 years.

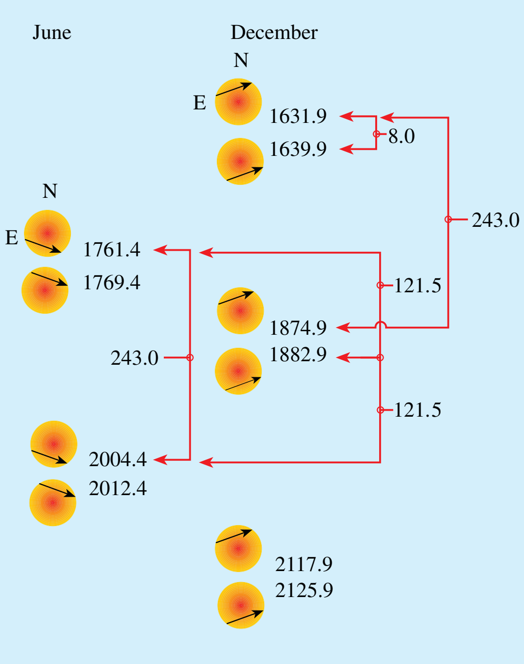

Figure 2. The dates of transits of Venus and the time intervals between transits. The thumbnail sketches show the path of Venus across the Sun.

Details of a transit

As can be seen in figure 3, a transit of Venus is analogous to an eclipse of the Sun by the Moon, except that the relative distances and angular scales are very different. Figure 3 is a plan view of the solar system, showing how the shadow of Venus will sweep past the Earth from east to west. Shown at the bottom are successive views of the Sun from Earth, showing how Venus can be seen against the disc of the Sun at various times. These should be read from right to left and the events t_1, t_2, t_3, t_4 correspond to the outer and inner tangential contacts of Venus's disc with the Sun's limb as Venus 'overtakes' the Earth. These are referred to in order of occurrence as: ingress, exterior and interior contact, and egress, interior and exterior contact. The positions of Venus relative to the Sun at these times are shown in figures 7 and 11. The actual times corresponding to the events t_1, t_2, t_3, t_4 depend on where the observations are made.

The fundamental plane

The transit is most easily understood in a reference frame that revolves with the Earth around the Sun.

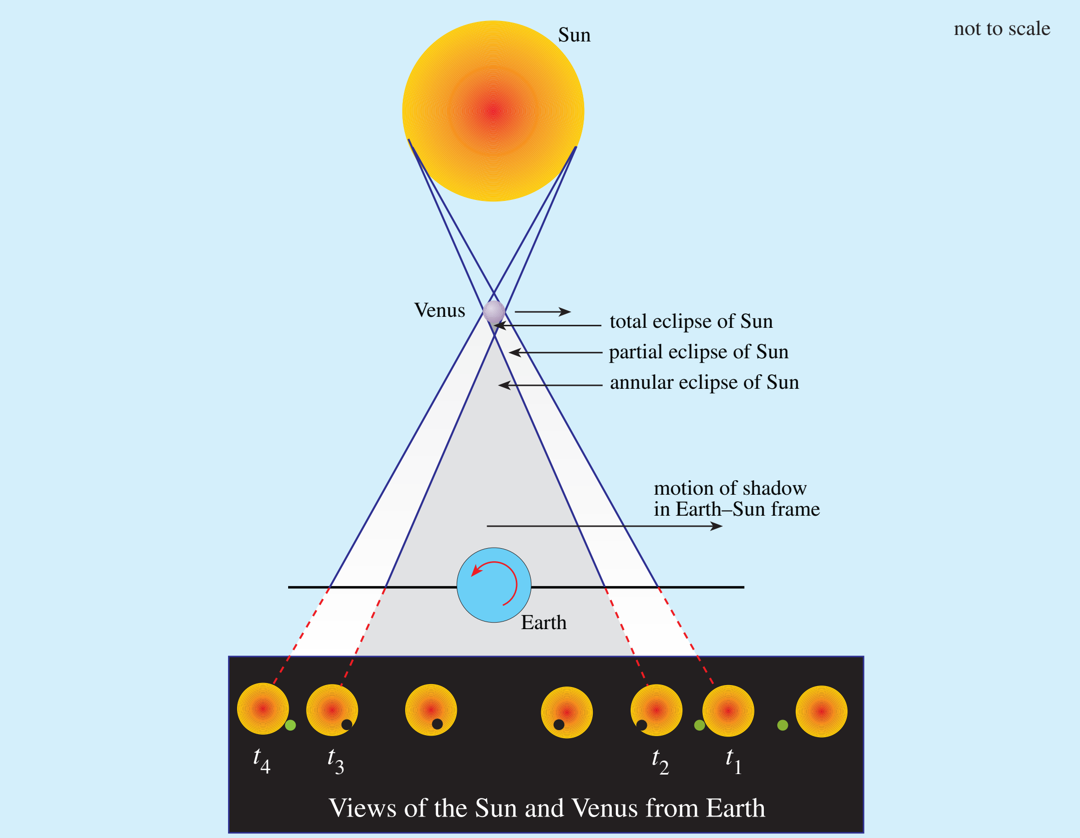

Figure 3. A plan view of the Sun, Venus and Earth, not to scale and in a frame where the Earth is stationary, showing the boundaries of the shadow created by Venus as it crosses the Sun. Views of Venus and the Sun at various positions within the shadow are shown at the bottom. Venus is invisible beyond the Sun's limb.

In this reference frame the Earth and Sun remain fixed and Venus passes between them. This can be understood by considering figure 1, where the arrows show their relative orbital motions. You may like to build a model to help you visualize this (see the Paper Plate article in Frontline).

Put a flat sheet on figure 3 at right angles to the paper on the straight line crossing the Earth, which is itself at right angles to the Sun–Venus–Earth line. If this is viewed from behind the Earth, looking towards the Sun, then you are looking at the *fundamental plane*. All movements and positions are projected onto this plane. Figure 4 is a view of the fundamental plane showing the Earth (which doesn't move) and the location of Venus's shadow at two times corresponding to the first (t_1) and last (t_4) contacts as observed from London. The positions of London and Pretoria projected onto the fundamental plane change during the transit because of the Earth's rotation.

The ecliptic is the plane of the Earth's orbit and therefore, viewed from Earth, it also marks the path of the Sun around the sky. Since the 2004 transit occurs 13 days before the summer solstice, the rotation axis and North Pole are mostly tilted towards the Sun with a residual tilt towards the East of only 5°.33 in the fundamental plane. Because this reference frame is rotating with the Earth around the Sun, Venus's shadow moves across the fundamental plane at an angle of $\gamma = 8°.49$ to the ecliptic. (This is greater than the 3°.394 inclination of the orbital plane, because the Earth's angular motion has been subtracted from that of Venus.)

Figure 5 shows, not to scale, a view of two observing sites on Earth that are both observing the transit. (They could be separated in either longitude or latitude or both.) Here we can see the relationship between the distance s in km between two observers, O1 and O2, on Earth and the

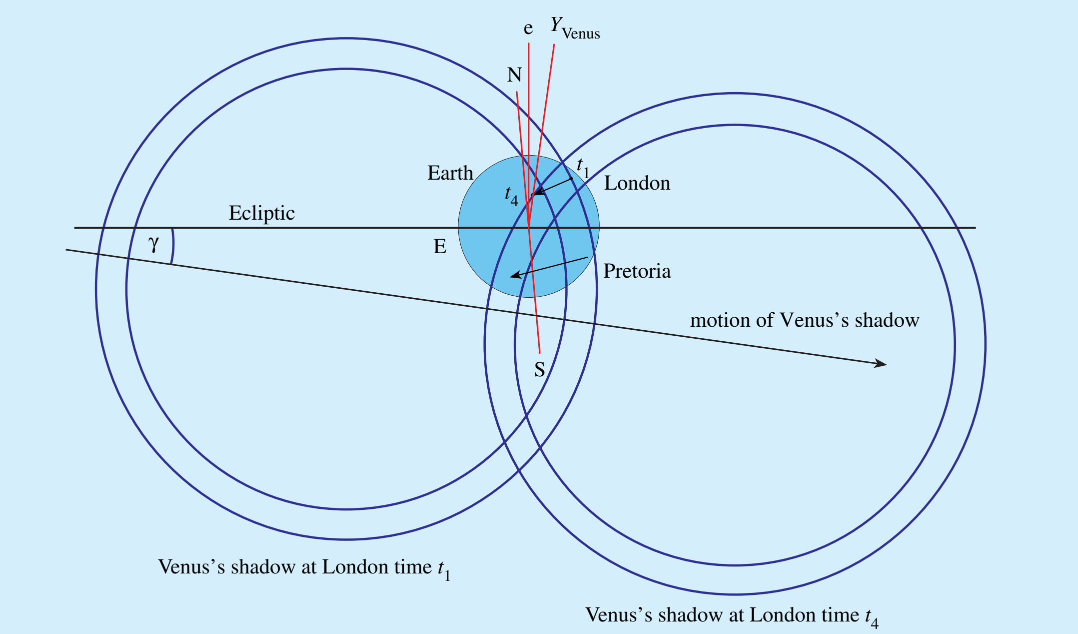

Figure 4. A view from outside Earth's orbit looking towards the Earth and the Sun. The plane of the paper is at right angles to the Earth–Venus–Sun direction and is called the fundamental plane. Imagine it is a screen mounted vertically at the bottom of figure 3. The large circles represent Venus's shadow drawn at two positions, corresponding to the first and last contact of Venus with the rim of the Sun, at times t_1 and t_4, as observed from London. Note the motion of London and Pretoria during the eclipse (from the tail to the head of their respective arrows), shown in more detail in figure 12. The directions N, e and Y_{Venus} denote the Y-axis of the three reference frames: on Earth, the ecliptic and in the direction of motion of Venus's shadow.

angular separation β in radians of their respective simultaneous views of Venus as it passes in front of the Sun. The Sun is at distance D from Earth. If the observation points on the Sun are a distance x apart:

$$\alpha = \frac{x}{d_1} = \frac{s}{d_2} \qquad \beta = \frac{x}{D}.$$

These can be rearranged to give the equation relating distance and angle that lies at the heart of all four methods:

$$D = \frac{d_1}{d_2}\frac{s}{\beta}. \tag{1}$$

The ratio d_1/d_2 can be found by observing the greatest angular separation of Venus from the Sun and using the geometry shown in figure 6, from which

$$d_1 = (d_1 + d_2)\sin\theta$$

which yields

$$\frac{d_1}{d_2} = \frac{\sin\theta}{1 - \sin\theta}.$$

The ratio can also be deduced from Kepler's third law, which can be written

$$\frac{A_{\text{Venus}}}{A_{\text{Earth}}} = \left(\frac{P_{\text{Venus}}}{P_{\text{Earth}}}\right)^{2/3}$$

where P_{Venus} and P_{Earth} are the orbital periods of Venus and the Earth and A_{Venus} and A_{Earth} are the semi-major axes of the orbits of Venus and the Earth. If their orbits were both circular then it is clear that

$$d_1 = A_{\text{Venus}} \qquad \text{and} \qquad d_1 + d_2 = A_{\text{Earth}}.$$

Because the orbits of both Venus and Earth are not quite circular, the value on 8 June 2004 given in the *Astronomical Almanac* is

$$\frac{d_1}{d_2} = 2.512.$$

Method 1: Distance to the Sun based on observations from at least two latitudes

When the transit is observed from different latitudes on Earth, Venus will appear to cross the

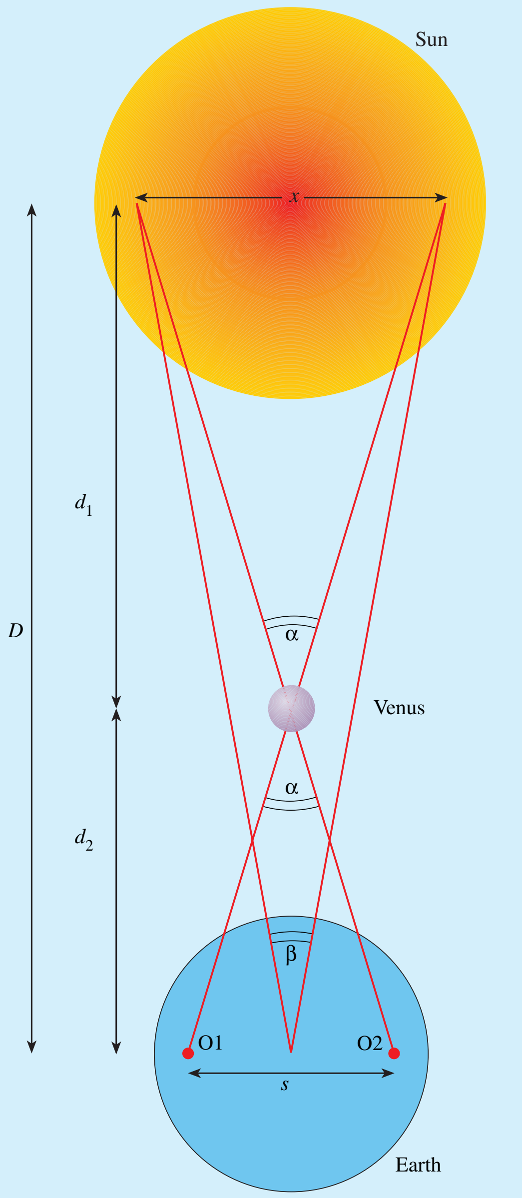

Figure 5. A plan view of the Sun, Venus and Earth, not to scale, showing the geometrical relationships between distance on the Earth, the distance to the Sun and angular distance on the Sun used to derive equation (1). The diagram also shows how the observers, O1 and O2, will each see Venus projected onto a different part of the Sun.

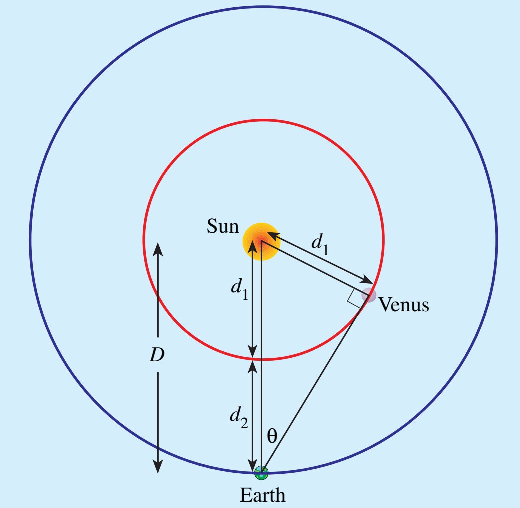

Figure 6. The geometry used to derive the relative size of the orbits of Venus and the Earth by observing the maximum angular distance of Venus from the Sun. Venus is shown as it will be in the sky before sunrise in early August 2004.

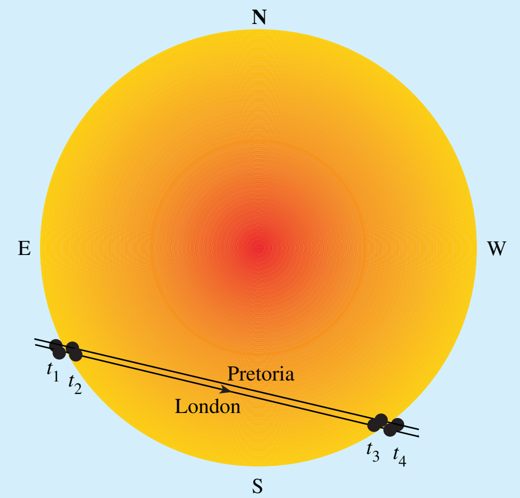

Figure 7. The track of Venus across the face of the Sun as seen from London and Pretoria. The size of the Sun, Venus and the separation of the two chords, which is 28 seconds of arc, are all to scale. From left to right, the four positions of Venus along each track correspond to tangential contacts, denoted ingress exterior and interior contact and egress interior and exterior contact. These are denoted by the events t_1, t_2, t_3 and t_4.

Sun along different chords. This is shown in figure 7 for the case of London and Pretoria, where the relative sizes of the Sun and Venus and the separation of the two chords are drawn to scale.

Close examination of figure 4 and also figure 12 shows that the projected separation of Pretoria and London in the fundamental plane, changes during

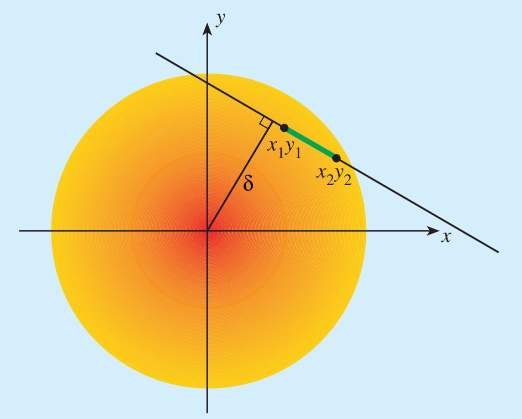

Figure 8. The geometry relating the direction of a chord to its minimum distance from the centre of a circle.

the course of the transit so that the chords shown in figure 7 are neither straight nor exactly parallel. However, they can be treated as such for the sake of simplicity. A chord can be characterized by its minimum angular distance δ from the centre of the Sun, illustrated in figure 8.

Figure 8 shows that it is not necessary to observe the entire chord to find the minimum distance from the centre. The chord can be specified by two observations, (x_1, y_1) and (x_2, y_2), recorded on successive images from the same site. From these observations it is possible to fit a straight line defined by

$$y = mx + c$$

where

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad c = y_1 - mx.$$

It can then be shown that the minimum distance δ is given by

$$\delta = \frac{c}{\sqrt{1 + m^2}}.$$

Although originally measured in pixels or mm, δ must be converted to an angle measured in radians by finding the angular scale of the image as outlined below.

The observations

Sets of observations must be made from at least two widely separated sites. At each site you need at least two images of Venus in transit, separated by sufficient time to define the chord. (A better result will be obtained with more observations, which will reduce the errors.) In reducing the data it is necessary to know the direction of N on each image and the angular scale.

If you use an equatorially mounted telescope N will always be in the same direction. After taking a picture, switch off the telescope drive for about a minute and take a second picture. The direction of drift of the image (to the west) defines the East–West direction. If the telescope is mounted in any other way it will be necessary to find N by drifting the image after each exposure. This is important as the direction of N in the field of a telescope in the UK will change by 40° during the transit.

The angular diameter of the Sun, which varies during the course of the year, can be found by a similar method, recording the time interval between the two exposures and assuming the Sun travels 360° around the Earth in 24 hours. (This is a good enough approximation although the ‘equation of time’, arising in part from the Earth’s elliptical orbit, means this is not quite true.)

The distance between the two observing sites

It is also necessary to find the distance s in kilometres between the two places projected onto the fundamental plane, shown in figure 4. This requires spherical trigonometry but a good approximation can be made, and more easily

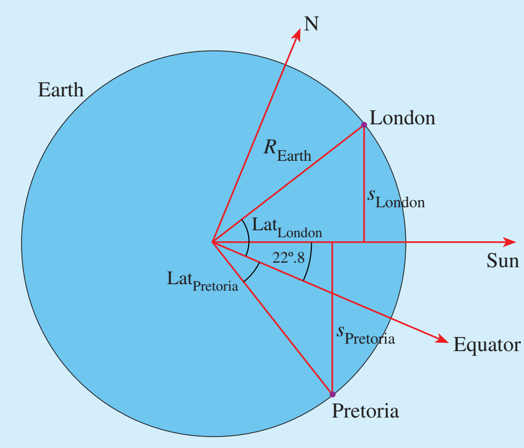

Figure 9. The simple geometry relating the distance between London and Pretoria to their latitudes measured from Earth’s equator and the direction of the Sun.

understood, by using the simplified relationship shown in figure 9, where London and Pretoria are taken as examples. Here R_{Earth} is the radius of the Earth, Lat is the latitude and 22°.8 is the projection of the inclination of the ecliptic. This angle varies between 0° and 23°.4 as the Earth moves from the equinox to the summer solstice.

$$s_{\mathrm{London}} = R_{\mathrm{Earth}} \sin(\mathrm{Lat}_{\mathrm{London}} - 22°.8)$$

$$s_{\mathrm{Pretoria}} = R_{\mathrm{Earth}} \sin(\mathrm{Lat}_{\mathrm{Pretoria}} - 22°.8)$$

$$s_{\mathrm{NS}} = s_{\mathrm{London}} - s_{\mathrm{Pretoria}}.$$

The North–South distance s_{NS} must be projected from the direction N onto the direction Y_{Venus} as indicated in figure 4. Y_{Venus} is at right angles to the direction of motion of Venus's shadow:

$$s = s_{\mathrm{NS}} \cos(5°.33 + 8°.49).$$

This method takes no account of longitude differences, and in the case of London and Pretoria underestimates s by about 7%. More accurate values can be found using the equations given in section 2 below.

Finding the distance to the Sun

The difference in the minimum angular distance δ measured at each site gives the angle β in equation (1). This is combined with the physical distance s obtained above to give the distance of the Sun:

$$D = \frac{d_1}{d_2} \frac{s}{\delta_{\mathrm{London}} - \delta_{\mathrm{Pretoria}}}.$$

Predictions show that the difference in δ between London and Pretoria is about 28 seconds of arc, half the diameter of Venus or about 1/60th the diameter of the Sun.

Method 2: Distance to the Sun based on timing the duration of the transit at one site

Inspection of figure 4 shows that the time it takes Venus to cross the Sun depends on two factors: firstly, the rate at which the shadow crosses the fundamental plane, which is the same all over the Earth, and secondly, the amount the observing site moves during the transit. Figure 4 shows that in London the transit ends slightly earlier than it would have done if the Earth had not rotated.

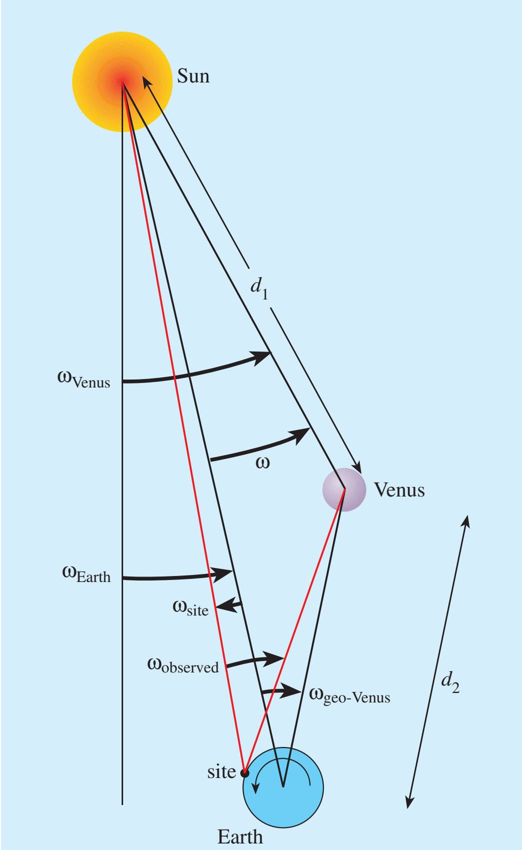

Figure 10. The relative positions of the Sun, Venus and the Earth after a short interval of time, as a consequence of their different angular velocities, which are defined in the text. The figure also shows the relationship between heliocentric and geocentric angular velocities, which are vectors, denoted by the bold lines.

It also shows that the effect is much greater for Pretoria, which is nearer the equator and moves further.

The advantage of this method is that it needs data from only one observing site, while the disadvantage is that it requires measurements of both the time taken to traverse the chord and the length of the chord. Because the angular motion of the observing site is so small in relation to the angular motion of the Earth and Venus and hence the shadow, the method is very sensitive to uncertainties in the angular velocities. To calculate these angular velocities, the relative scale and shape (eccentricity) of the orbits of Venus and the Earth must be known. For example, the transit seen from Pretoria takes about 6.2 hours but ends 6 minutes earlier as a result of the Earth's rotation. The 6 minutes is the time taken for the shadow to traverse the distance moved by Pretoria during the transit and from this the distance of the Sun is calculated. A small error in the angular velocities of Venus and the Earth results in a proportionally large error in the estimated residual.

As a matter of interest, the uniform daily rotation of the observing station does not map on to the fundamental plane as a uniform motion, so the observed rate of motion of Venus across the Sun is not constant.

Figure 10 shows the definition of the various angular velocities and the derivation of the equation

$$d_2\omega_{\text{observed}} = d_1[\omega_{\text{Venus}} - (\omega_{\text{Earth}} + \omega_{\text{site}})]. \quad (2)$$

The observed angular velocity of Venus, ω_{observed}, is different for each observing site. The angular velocity of Venus seen from the centre of the Earth, called the geocentric angular velocity, is given by

$$\omega_{\text{geo-Venus}} = \frac{d_1}{d_2}(\omega_{\text{Venus}} - \omega_{\text{Earth}}). \quad (3)$$

These angular velocities are vectors so that we must project ω_{Earth} and ω_{site} onto the direction of motion of Venus's shadow, as indicated below.

The required observations and derivation of the observed angular velocity

As mentioned above, there are four critical events: t_1, t_2, t_3, t_4, corresponding to the moments when Venus appears tangential to the limb of the Sun as shown in figure 7, for which timings are required. The first of these will be extremely difficult to observe for all but the most experienced observers, who know in advance exactly where and when Venus will first touch the limb.

During the course of the transit it is necessary to determine the chord, and from this find the minimum distance from the centre of the Sun as well as the diameter of Venus. All these must be converted to angular distances, as described in the previous section. The appropriate lengths, corresponding to the different events, are illustrated in figure 11 and given by the relationship

$$L_1^2 = (R_{\text{Sun}} - R_{\text{Venus}})^2 - \delta^2 \quad \text{(for the interval } t_2 \text{ to } t_3\text{)}$$

$$L_2^2 = (R_{\text{Sun}} + R_{\text{Venus}})^2 - \delta^2 \quad \text{(for the interval } t_1 \text{ to } t_4\text{)}$$

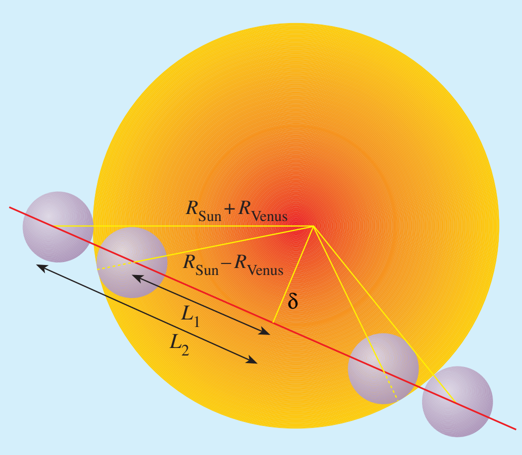

Figure 11. The geometry relating the length of the path of Venus across the Sun to the radius of the Sun, R_{Sun}, and Venus, R_{Venus}, at the four tangential contacts.

where δ is found by the method given in section 1 above. The observed angular velocity is then simply

$$\omega_{\text{observed}} = \frac{2L_1}{t_3 - t_2} \quad \text{or} \quad \omega_{\text{observed}} = \frac{2L_2}{t_4 - t_1}. \quad (4)$$

Here, to avoid a proliferation of subscripts, t_1, t_2, t_3, t_4 refer to the times of the events.

The geocentric angular velocity of Venus

The geocentric angular velocity refers to the angular velocity with respect to the centre of the Earth and is derived from the angular velocity about the Sun (heliocentric angular velocity). The centre of the Earth is identical to the origin of the coordinate system in the fundamental plane, shown in figure 4.

The mean heliocentric angular velocity of a planet, with orbital period P_{Planet}, about the Sun is

$$\omega_{\text{Mean}} = \frac{360°}{P_{\text{Planet}}} \quad \text{day}^{-1}.$$

However, it is necessary to know the angular velocity to a greater accuracy as it varies during the course of the year, due to the eccentricity of the orbits of Venus and the Earth.

The velocity V of a planet having semi-major axis A at true distance D from the Sun, where $\mu = G(M_{\text{Sun}} + M_{\text{Planet}})$, is given by the relation

$$V^2 = \mu\left(\frac{2}{D_{\text{Planet}}} - \frac{1}{A_{\text{Planet}}}\right).$$

By definition $V = \omega D_{\text{Planet}}$. This leads to the relation for the angular velocity corresponding to the distance D:

$$\omega_D = \omega_{\text{Mean}} \frac{A_{\text{Planet}}}{D_{\text{Planet}}} \sqrt{\frac{2A_{\text{Planet}}}{D_{\text{Planet}}} - 1}.$$

Here, all the distances appear as ratios, so we can set the distance A of the Earth from the Sun equal to 1.000, which we call the astronomical unit. (It is this distance that we are trying to find in kilometres and which would be necessary if we wished to know the velocity V in km s^{-1}.) The parameters for 8 June are shown in table 2.

Table 2. Parameters for 8 June.

	Venus	Earth
A	0.7233	1.000
D	0.726	1.015

Since we are working in an Earth–Sun rotating reference frame we must subtract the angular velocity of the Earth from the angular velocity of Venus. Resolving velocities along and at right angles to the ecliptic, where i is the inclination of Venus's orbit to the ecliptic, the following formulae apply at the nodes:

$$\omega_{\text{ecliptic } X} = \omega_{\text{Venus}} \cos i - \omega_{\text{Earth}}$$
$$\omega_{\text{ecliptic } Y} = \omega_{\text{Venus}} \sin i.$$

The inclination γ of the Venus shadow path to the ecliptic in the Earth–Sun rotating frame shown in figure 4 is

$$\gamma = \tan^{-1}\left(\frac{\omega_{\text{ecliptic } Y}}{\omega_{\text{ecliptic } X}}\right).$$

The total angular velocity is

$$\omega^2 = \omega^2_{\text{ecliptic } X} + \omega^2_{\text{ecliptic } Y}.$$

These are all heliocentric angular velocities. As seen from the Earth the geocentric angular velocity of Venus is

$$\omega_{\text{geo-Venus}} = \omega \frac{d_1}{d_2}.$$

The discrepancy between the value used in the *Astronomical Almanac* and the value calculated here (see table 3) may arise from failing to account for the motion of the Earth about the centre of gravity of the Earth–Moon system.

Table 3. The geocentric angular velocity of Venus (8 June 2004).

	$\omega_{\text{geo-Venus}}$ (arcsec s^{-1})	$\omega_{\text{geo-Venus}}$ (rad s^{-1})
Deduced from data above	0.066 78	3.237×10^{-7}
Used in the *Astronomical Almanac*	0.066 81	3.239×10^{-7}

The motion of the observing site in the fundamental plane

In order to find the heliocentric angular velocity of the observing site, it is necessary to find the difference in position of the observing site in the fundamental plane, between the start and end of the transit, projected onto the direction of motion of the shadow. As mentioned above, this requires the use of spherical geometry. The equations are given below without derivation. The required input data are:

- The latitude and longitude of the observing site in degrees: *Lat*, *Long*
- The times in UTC, in hours: t_{start}, t_{end}
- The equation of time in hours on 8 June: $Eqtime = 0.023$ h
- The radius of the Earth: $R_{\text{Earth}} = 6371$ km
- The angle between the ecliptic and the direction of shadow motion: $\gamma = 8°.486$.

Then calculate

$$\alpha = Long + 166.830 + 360\left(\frac{t_{(\text{UTC})} - 12 + Eqtime}{24}\right).$$

The numerical constants in these equations depend only on the inclination of the ecliptic to the equator and the ecliptic longitude of the Sun on 8 June. The angle α is the difference in longitude between the place of observation and the place where the Sun is vertically overhead. It is not illustrated.

Find the angular y coordinate in the ecliptic system:

$$E_y = \sin^{-1}(0.918 \sin Lat + 0.397 \cos Lat \cos \alpha).$$

Find the angular x coordinate in the ecliptic system

$$E_x = 12°.12 \times \cos^{-1}\left(\frac{0.397 \sin Lat - 0.918 \cos Lat \cos \alpha}{\cos E_y}\right).$$

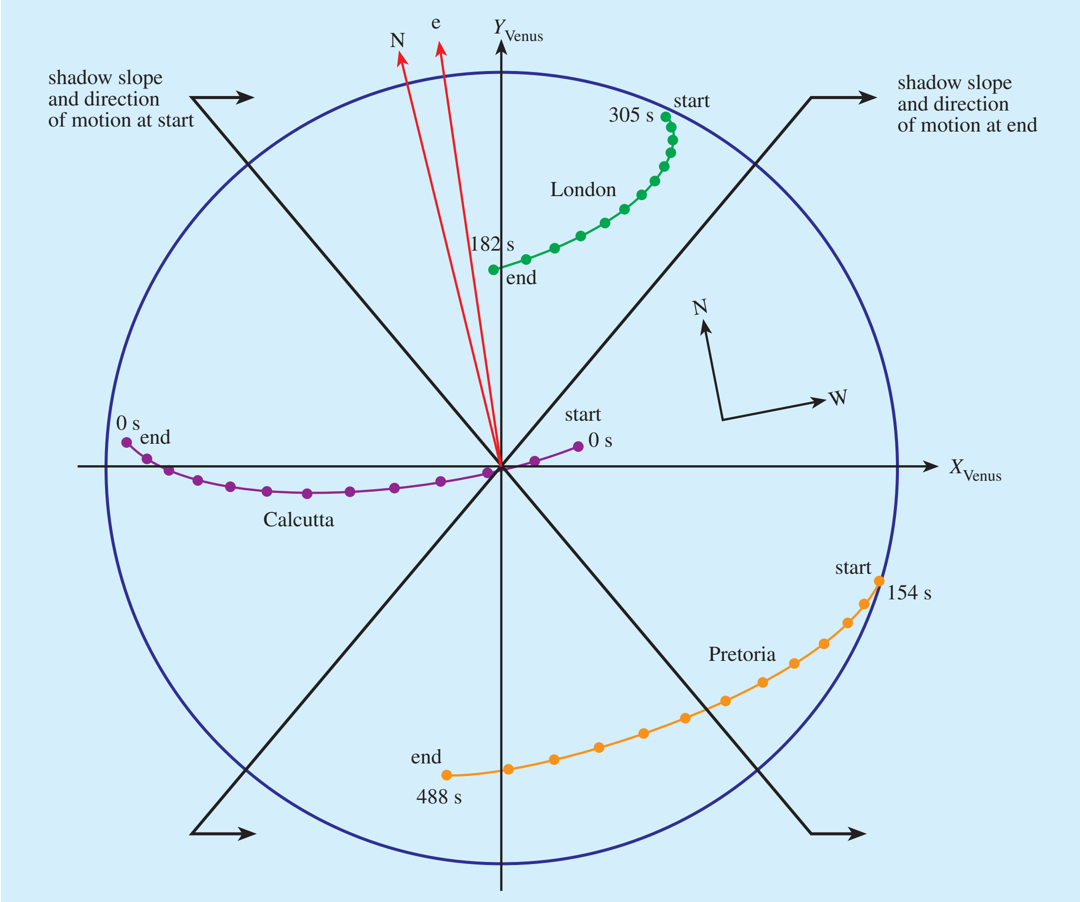

Figure 12. A detail of the fundamental plane shown in figure 4, rotated so that the direction of Venus's shadow lies along the x axis, denoted X_{Venus}, showing the trajectories of London, Calcutta and Pretoria. The symbols mark half hour intervals from the start. The last symbol marks the end of the transit, making the last interval slightly longer. The circle, radius R_{Earth}, represents the Earth and the directions of North and the ecliptic pole are shown. Also shown are the times in seconds at the start and end of the transit relative to Calcutta and the orientation of the shadow edge. The shadow edge is shown as a line, tangent to the limb of the Sun. In reality it is part of a circle with radius 44 times the radius of the Earth.

Also evaluate

$$E_x = 12°.12 - \sin^{-1}\left(\frac{\cos Lat \sin\alpha}{\cos E_y}\right).$$

Compare the two values of E_x and determine which is correct, noting that E_x must change uniformly. The ambiguity arises because computed values of $\sin^{-1}$ are only returned between $\pm 90°$ and $\cos^{-1}$ between 0° and 180°.

Project the ecliptic coordinates onto the fundamental plane:

$$X_e = R_{Earth} \cos E_y \sin E_x$$

$$Y_e = R_{Earth} \sin E_y.$$

Rotate these onto the Venus coordinate system:

$$X_{Venus} = -(X_e \cos\gamma + Y_e \sin\gamma)$$

$$Y_{Venus} = Y_e \cos\gamma - X_e \sin\gamma.$$

The total projected distance travelled by the observer in the direction of motion of Venus's shadow is then

$$s = X_{Venus}(t_{end}) - X_{Venus}(t_{start}).$$

The heliocentric angular velocity of the observing site

The heliocentric angular velocity of the observing site in radians is simply the velocity of the site in the fundamental plane divided by the unknown distance D to the Sun:

$$\omega_{\text{site}} = \frac{s}{D(t_{\text{end}} - t_{\text{start}})}. \qquad (5)$$

Note this is always negative, except for sites north of latitude 68°, which view the transit over the North Pole, when it is positive.

Finding the distance to the Sun

Substituting equations (3), (4) and (5) into equation (2) and working in radians, we can solve for the distance D in kilometres:

$$D = \frac{d_1}{d_2} \frac{s}{[(t_{\text{end}} - t_{\text{start}})\omega_{\text{Venus}} - 2L]_{\text{radians}}}.$$

Compare this equation with equation (1) and note the similarity.

Method 3: Distance to the Sun based on absolute timings obtained from at least three sites

Figure 12 gives an enlarged view of the fundamental plane showing the tracks of London, Pretoria and Calcutta during the course of the transit. As the shadow of Venus sweeps across the fundamental plane, the start and end of the eclipse will be seen at different times at each of the three places. Relating the known angular velocity of the shadow to the measured speed of the shadow provides a method of determining the distance of the Sun, provided of course that the latitude and longitude of the places are known and all places record the times of the events in UTC. The difference in start and end times is quite large and easy to measure. Predictions, given in the *Astronomical Almanac*, show that the eclipse will start in Calcutta about 305 seconds before London and 154 seconds before Pretoria.

It is necessary to combine data from a minimum of three sites because both the speed and the orientation of the shadow must be found. We approximate the shape of the shadow by a tangent to the Sun's limb. The true shape is an arc reproducing the curvature of the Sun's limb. From figure 4 it is clear that the tangent will have a negative slope at the start of the eclipse and a positive slope of the same magnitude at the end of the eclipse. The tangent calculated from the predictions is drawn in figure 12.

Observations

Timings of only some of the four events t_1, t_2, t_3, t_4 are required, but data are needed from at least three separate sites for each chosen event. If local times are used they must be converted to UTC. This method has the advantage that data can be used from sites that only see either the start or end of the transit, due to either cloud or unfavourable longitude.

Method

We will assume that the positions of all the sites are found in the Venus coordinate system in the fundamental plane by the methods given above. Assume that at each site the same eclipse event is observed. The nth site has the data set

$$t_n, x_n, y_n$$

where the x direction in the Venus coordinate system is in the direction of motion of the shadow in the fundamental plane. We assume a linear relation between the three coordinates for each of the places, denoted l, m, n, and write

$$\begin{aligned} t_l &= ax_l + by_l + c \\ t_m &= ax_m + by_m + c \\ t_n &= ax_n + by_n + c. \end{aligned}$$

We must find the three unknowns a, b and c. These are given by the three equations

$$a = \frac{\begin{matrix} t_l & y_l & 1 \\ t_m & y_m & 1 \\ t_n & y_n & 1 \end{matrix}}{\begin{matrix} x_l & y_l & 1 \\ x_m & y_m & 1 \\ x_n & y_n & 1 \end{matrix}} \qquad b = \frac{\begin{matrix} x_l & t_l & 1 \\ x_m & t_m & 1 \\ x_n & t_n & 1 \end{matrix}}{\begin{matrix} x_l & y_l & 1 \\ x_m & y_m & 1 \\ x_n & y_n & 1 \end{matrix}}$$

$$c = \frac{\begin{matrix} x_l & y_l & t_l \\ x_m & y_m & t_m \\ x_n & y_n & t_n \end{matrix}}{\begin{matrix} x_l & y_l & 1 \\ x_m & y_m & 1 \\ x_n & y_n & 1 \end{matrix}}.$$

This can be resolved:

$$a = \frac{t_l(y_m - y_n) - y_l(t_m - t_n) + (t_m y_n - t_n y_m)}{x_l(y_m - y_n) - y_l(x_m - x_n) + (x_m y_n - y_m x_n)}$$

$$b = \frac{x_l(t_m - t_n) - t_l(x_m - x_n) + (x_m t_n - x_n t_m)}{x_l(y_m - y_n) - y_l(x_m - x_n) + (x_m y_n - y_m x_n)}$$

$$c = \frac{x_l(y_m t_n - y_n t_m) - y_l(x_m t_n - x_n t_m) + t_l(x_m y_n - x_n y_m)}{x_l(y_m - y_n) - y_l(x_m - x_n) + (x_m y_n - y_m x_n)}.$$

To find the meaning of a, b and c consider the equation

$$t = ax + by + c.$$

When t is constant this equation describes the locus of constant time, which is also the locus of the tangent to the limb, the edge of the shadow. The equation can be written in the more familiar form

$$y = -\frac{a}{b}x - \frac{c+t}{b}$$

where $-a/b$ is the 'slope'. Differentiating the equation $t = ax + by + c$ with respect to time t, and noting that the direction of travel is along the x-axis, we find that the velocity of the shadow is simply $1/a$.

Equation (1) can be rewritten as

$$D = \frac{d_1}{d_2}\frac{\mathrm{d}S/\mathrm{d}t}{\mathrm{d}\beta/\mathrm{d}t}$$

where

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{1}{a} \qquad \frac{\mathrm{d}\beta}{\mathrm{d}t} = \omega_{\mathrm{Venus}}.$$

to give the distance D.

The same method can be applied at the end of the transit and the results compared. Computation is simplified if one of the sites is chosen as a zero point for the times and positions are expressed in units of the Earth's radius.

Method 4: Distance to the Sun based on the time taken to transit the Sun from at least three places

It is apparent from figure 7 that the time taken for Venus to cross the Sun will depend on the length of the chord. This increases as Venus is viewed from more southerly latitudes simply due to the curvature of the Sun's limb. Another way of understanding this is by considering how the shadow moves across the fundamental plane from the beginning to the end of the transit, as shown in figure 12. The length of the transit will also depend on how far the sites move during the transit (as explained in sections 2 and 3 above). The methodology for solving the problem is very similar to that outlined in previous sections.

Observations

The time taken for the transit must be measured from at least three sites. Unlike method 3, this requires the beginning and end of the transit to be visible from each site. This limits the sites that can be used and means the weather must be clear at the start and finish.

Method

The positions of all three sites, denoted, l, m, n, are found in the fundamental plane, as described in section 2, at the beginning and end of the transit. For each site we require the duration of the transit Δt, the distance moved in the fundamental plane Δx and the average position y. As in section 3, we assume a linear relation between these three and ignore terms relating to the change in y during the transit.

$$\Delta t_l = a\Delta x_l + by_l + c$$
$$\Delta t_m = a\Delta x_m + by_m + c$$
$$\Delta t_n = a\Delta x_n + by_n + c.$$

These can be solved by the method given in section 3, once again giving the value $1/a$, the velocity of the shadow, which can be used to find the distance of the Sun. The constant c is the geocentric transit time, which is the time of transit observed from the centre of the Earth. The slope, which in the previous section gave us the slope of the shadow, here gives us the dependence of Δx on the y coordinate, a not very interesting parameter as it depends on both shadow slope and latitude of the observing site.

Making observations

When advocating his method in 1716, Halley noted that timing the transit with an accuracy of one second of time would give a very good positional accuracy (0.07″) for Venus. This was far more accurate than could be achieved by any other method then available. However, in practice the method proved to be more difficult than anticipated. The greatest source of error was in timing exactly when the image of Venus was tangential to the edge of the Sun. This was partly due to the observers using small aperture telescopes producing diffraction effects that gave the impression that Venus was attached to the limb of the Sun by a meniscus like a 'black drop'.

Table 4. The parameters used and how to obtain them.

Parameter	Method	Value
Before the transit		
Radius of the Earth	Measure the length of the shadow cast by a vertical rod of known length, at two places separated by a known distance along the circumference of the Earth.	$R_{\text{Earth}} = 6371$ km
Rotation period of Earth	Find the time interval between identical shadow directions cast by the Sun from one day to the next. Allow for the equation of time.	24 h
Latitude and longitude of site of observation	Latitude: Observe the elevation of a star of known declination when it is due south. Longitude: Compare the time when a star lies due south with the time it does so at Greenwich using the same time system (UTC) at both places.	*Lat, Long*
The angular radius of the Sun	Record the image of the Sun twice within about 90 seconds. Measure the angular diameter of the Sun's image in terms of the separation of the two images, bearing in mind that the Sun moves through approximately 360° in 24 hours.	$R_{\text{Sun}} = 945.4''$
The orbital (sidereal) period of Venus and Earth	By careful observation of Venus's motion through the night sky over a period of years.	$P_{\text{Venus}} = 224.695$ d $P_{\text{Earth}} = 365.256$ d
The relative semi-major axes of the orbits of Venus and Earth	Deduced from observations of the maximum angular separation of Venus from the Sun over a period of years.	Earth = 1.000 Venus = 0.723
The relative orbital radii of Venus and Earth on 8 June	*Values for 8 June 2004* d_1 = radius of Venus's orbit d_2 = radius of Earth's orbit minus radius of Venus's orbit Only the ratio can be found.	$d_1 = 0.726$ AU $d_1 + d_2 = 1.015$ AU $d_1/d_2 = 2.512$
Inclination of Venus's orbit to the ecliptic	Found by careful observation of Venus's motion in the night sky and its deviation from the ecliptic.	$i = 3.394°$
During the transit		
The times of the events t_1, t_2, t_3, t_4 when Venus is tangential to Sun's limb	By a group observing a projected image, or by a single observer looking through a telescope with a suitable filter in front of the telescope.	Times of events t_1, t_2, t_3, t_4
The angular length of the chord described by Venus as it crosses the face of the Sun and the time it takes to cross	From a minimum of two images find the length of the chord and its minimum distance from the centre of the Sun, in angular units. Find the direction of North on the image. Record time and measure position on images to find the angular velocity.	L_1, L_2 δ ω_{observed}
Radius of Venus	Measure the angular radius of Venus relative to the angular radius of the Sun.	$R_{\text{Venus}} = 28.88''$
Result: the value of *D*	Distance of Sun = 1 AU = 149 597 870.66 km. The Sun–Earth distance on 8 June 2004 is 1.015 AU.	

The 'black drop' can be seen by bringing your thumb and forefinger together, in front of a bright background, with your fingers close enough to your eyes to be out of focus, This effect will not be a problem with the larger aperture high-quality telescopes available today, provided that suitable filters are mounted in front to protect the telescope and the observer.

The extent to which individual schools can make measurements depends on the equipment they have available. The ideal observing set-up is an equatorially mounted telescope with suitable filters mounted in front and a digital camera having sufficient scale or choice of scales to measure the diameters of both the Sun and Venus as well as to provide images timed to the nearest second, from which to measure the length of the chord and the angular velocity. Digital image processing software will also be required to complete the task. Photographs can be measured by hand, or scanned and measured digitally.

Few amateur astronomers and even fewer schools will be so well equipped. However, with relatively simple projection equipment, such as a pair of binoculars mounted on a stand projecting an image onto a card, several people can estimate the times of the two interior tangential contacts. As mentioned above, these vary significantly from different parts of the world. Within the UK, ingress will be seen about 13 seconds earlier in Glasgow than London. In South Africa, ingress will not be visible from Cape Town but egress will be seen about 75 seconds later than in Pretoria. So it should be possible for schools, even within the UK and South Africa to plot the motion of the shadow. The European Southern Observatory (ESO) has agreed to coordinate and reduce observations from around the world so that any accurate timing observations can contribute to their grand set of data and solution. Details of their website are given below.

At the very least, observing the transit will be an opportunity to see the solar system in motion, to confirm that Venus is round, to note the contrast between the 'black' silhouette of Venus and the dark grey of any sunspots that may be present and, given the relative distances of Venus and the Sun, to estimate their relative sizes.

Summary

Table 4 summarizes the steps used to determine the Earth–Sun distance from the transit of Venus and also provides a convenient listing of the parameters used and how in principle they might be obtained.

Useful websites and resources

- A very simple and seemingly safe solar projection system has been developed by the Observatoire de la Cote d'Azur and can be seen at www.solarscope.org.
  (See Reviews on page 305 of this issue.)
- Timing details can be found in the *Astronomical Almanac*.
- The European Southern Observatory is offering to coordinate observations. See their website at www.eso.org/outreach/eduoff/edu-prog/vt-2004/
- The University of Central Lancashire, close to the home of Jeremiah Horrocks, is developing outreach resources, which can be found at www.transit-of-venus.org.uk/
- A site with interesting details of South African transit observations can be found at www.saao.ac.za/~wpk/tov1882/tovwell.html
- A popular introduction with interesting historical material on transit expeditions can be found in the book by Maunder and Moore (1999).

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